6.1 Wavelengths and trig graphs

Overview

This section is intended to develop a deeper understanding of the links between mathematics and science as they apply to waves.

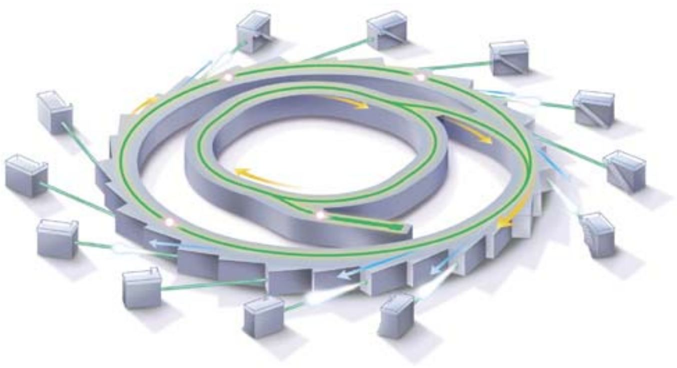
In mathematics the topic of wave motion is primarily based around the study of trigonometric functions. Consequently the focus in the first activity is an investigation of the function $y = a\sin(bx + c) + d$. Students are guided to discover the effects of the coefficients a, b, c and d on a graph.

The wave focus in science is often on the electromagnetic spectrum and its applications. Therefore, we will follow the mathematical investigation by exploring the interface between a mathematical view of a wave as a stationary function graphed on the Cartesian plane, and investigate the scientific view of a wave as a disturbance in space-time manifesting as the electromagnetic spectrum.

The mathematics of the sine function

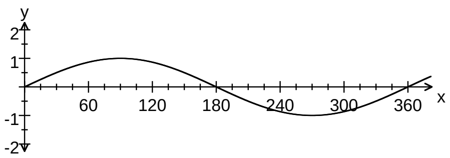
The primary output of a synchrotron is electromagnetic waves. The synchrotron can produce many different wavelengths and intensities.

The acceleration imposed on electrons as they encounter magnetic fields produces electromagnetic waves. Their wavelength, frequency and intensity are controlled by a number of factors, including the size of the acceleration and how suddenly it is applied.

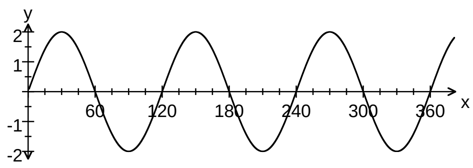


Diagrammatic representation of the beamlines of the Australian Synchrotron
Image courtesy: Australian Synchrotron, State of Victoria

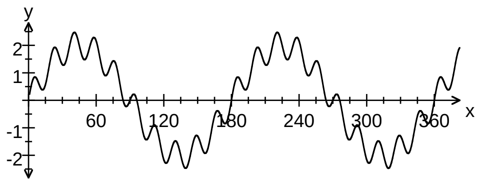
Viewed from a mathematical view point any electromagnetic wave can be modelled using a sine curve or a combination of multiple sine curves. Examples of sine waves of varying complexity are:



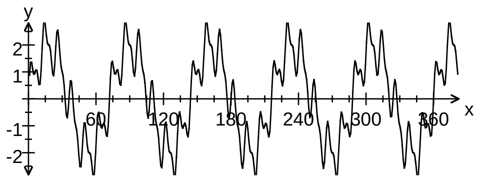
$y = \sin(x)$



$y = 2\sin(3x)$



$y = 2\sin(2x) + 0.5\sin(20x)$



$y = 2\sin(5x) + 0.8\sin(30x) + 0.4\sin(60x)$

In addition to electromagnetic waves, many other situations can be modelled and better understood using sine curves. These include:

- Financial information - trying to predict the future behaviour of the stock market.
- Earthquakes - analysing records of seismic waves to predict quakes.
- Heartbeats - predicting problems by analysing the heart's electrical behaviour.

Almost anything that exhibits regular cyclic behaviour can be modelled in this way.

The maths/science interface

Overview

In this activity students will investigate how the scientific terminology of amplitude, frequency, wavelength and phase-shift relate to the values of *a*, *b*, *c* and *d* from part 1.

It should include the difference between wavelength in trigonometric multiples (maths) and metric units (science); and different types of waves.

In terms of radio stations the following information on bands is given. Please note: these may not correspond to Australian band widths.

- FM band 88 to 108 MHz
- Long wave 148.5 to 283.5 kHz
- Medium wave 530 to 1710 kHz
- Short wave 3000 to 30000 kHz
- Mobile phone 870 to 2170 MHz (on 3 or 4 bands)

It is expected that this activity will use a number of programmed tools that illustrate various concepts. These might include:

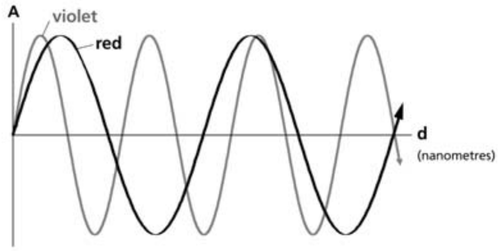
- A tool to illustrate the electromagnetic spectrum. By providing a slider, the user can move along the spectrum with information and illustrative pictures appearing as they do so.
- A tool to provide transition from the mathematical way of regarding the sine wave as a frozen function to the scientific way of regarding it as something moving past the observer.
- A tool to analyse the behaviour of $y = a\sin(bx + c) + d$. This might be a PC-based graphing package or a graphical calculator.

This activity

In the previous activity your students considered waves from the mathematical perspective of the equation $y = a\sin(bx + c) + d$. In science classes waves are considered in contexts such as sound waves and light waves. Waves are formed when disturbances are propagated from place to place in a regular and organised way. The most familiar waves are surface waves that travel on water, but sound and light also exhibit wave-like properties.

In a simple wave, the disturbance oscillates periodically with a fixed frequency and wavelength. Longitudinal waves, such as sound, require a material medium through which to travel, while electromagnetic waves such as light do not require a medium and can be propagated through a vacuum. The important factor about a wave is the property of energy transfer.

Wavelength



Light waves come in many wavelengths. Wavelength is the distance between any two corresponding points on the wave, for example successive peaks. The wavelengths of visible light range from 400 to 650 nanometres (nm = 10^{-9} m).

However, the full range of wavelengths included in the definition of electromagnetic radiation extends from 10^{-15} metres (gamma rays) to hundreds of metres (radio waves).

Visible light is only one small section of the electromagnetic spectrum.

Frequency

Light waves also come in many frequencies. Frequency is the number of waves that pass a point in space during any time interval, usually one second. It is measured in units of cycles (waves) per second, or Hertz (Hz). The frequency of visible light is referred to as colour, and ranges from 460 trillion Hz (seen as red) to 750 trillion Hz (violet).

The amount of energy in a light wave is proportionally related to its frequency: high frequency light has high energy; low frequency light has low energy. Thus gamma rays have the most energy, and radio waves have the least. Of visible light, violet has the most energy and red the least.

Light not only vibrates at different frequencies, it also travels at different speeds. Light waves move through a vacuum at their maximum speed (≈300000 kilometres per second). This means light is the fastest known phenomenon in the universe. Light waves slow down when they travel inside substances, such as air, water, glass or diamond