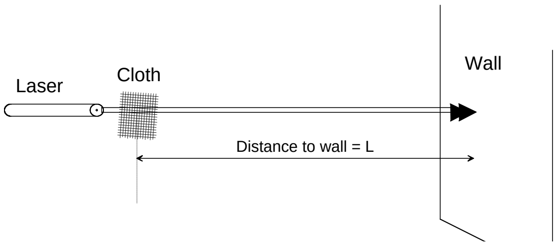
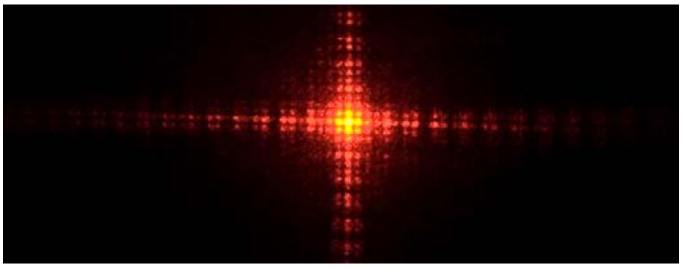
7.2 Measurement using interference effects

Measuring very small distances using a laser is quite easy. This simple process can be used to measure spacing between threads in cloth or the width of a human hair.

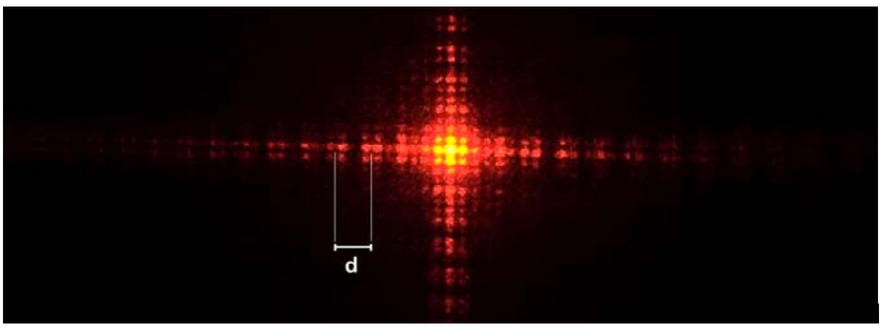


1. Position your mesh or cloth sample a measured distance from a white wall or screen.
2. Shine the laser light through the cloth onto the wall.
3. An interference pattern will form on the wall.



4. Measure the distance between the bright dots of the pattern.
5. The spacing w between threads in the weave can now be found using the formula $w = \frac{\lambda L}{d}$, where L is the distance (in metres) to the screen, d is the separation to the first dot (in metres) and λ is the wavelength of the laser light, generally 6.5×10^{-7} m.

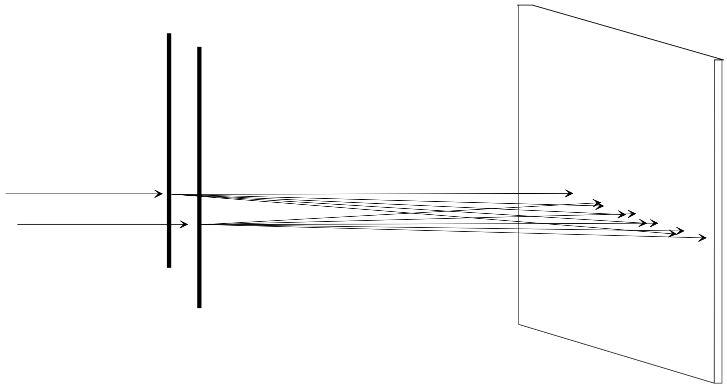
   For example, if the distance to the wall is two metres and the dot separation is 2.3 centimetres (0.023 meters), then w = 6.5 x 10^{-7} × 2 / 0.023 = 5.65 x 10^{-5} metres or 56.5 micrometres as the thread separation.
6. A similar effect can be used to measure the thickness of very small objects, such as hairs, but there are two differences:
   - The distance measured in the interference pattern is between the first two dark areas on either side of the central bright dot.



- The formula used is $w = \frac{2\lambda L}{d}$.

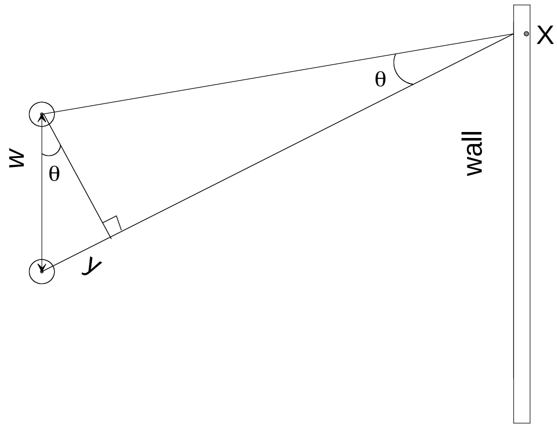
Why does it work?

Depending on your students' ability and science background, you can decide how much of the following explanation to share with your students. The average middle high school student is unlikely to have sufficient background.



Two adjacent threads are w metres apart. As light shines past them it is diffracted by the threads in the cloth.

The illustration on the right (view from above) shows one particular spot (X) identified on the screen. The lines pointing towards X represent light falling on the screen from two adjacent threads.



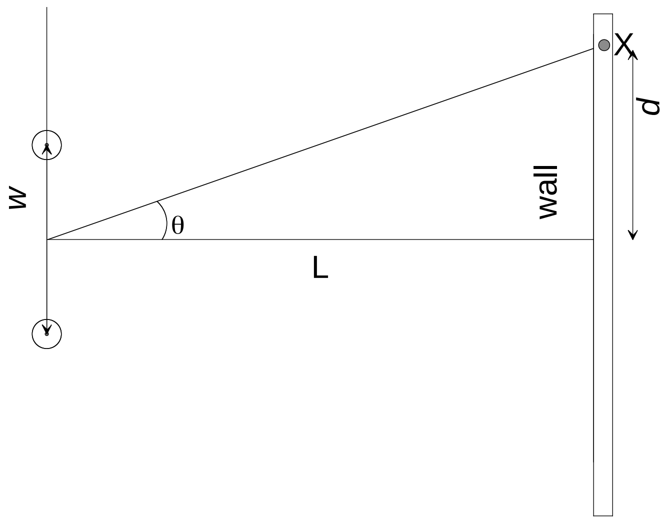
In this diagram y, the difference in distances to X, is given by $y = w\sin\theta$.

The distances from the two threads to X will be different due to where their diffracted light overlaps on the screen. This results in interference, as the two beams combine. The outcome of this is light will be brighter (constructive interference) in some places and darker (destructive interference) in others depending on the relative distance from each thread to X.

To create a bright spot *both* waves must be at a crest because adding these will make the resultant light brighter. Meaning the difference between the two distances, which is approximately y, must be a multiple of the wavelength λ.

Thus $y = w\sin\theta$ becomes $n\lambda = w\sin\theta$, where $n = 1, 2, 3\ldots$ We assume the point X is the location of the first bright spot and thus $n = 1$ and $\lambda = w\sin\theta$.

The distance from the thread to the wall is L.



If the line labelled L is the beam of the laser and the point X is the point where the first bright spot appears (see previous page) then we call this separation d.

The angle θ in this diagram is not the same as the value in the previous diagram, but is close to it. For the purposes of approximation we can assume these values for θ are the same.

This gives the result $\tan\theta = \frac{d}{L}$. Again using an approximation that for very small values of θ, $\tan\theta \approx \sin\theta$, we get $\sin\theta \approx \frac{d}{L}$.

If we now substitute this formula for $\sin\theta$ into our previous equation $\lambda = w\sin\theta$ we obtain $\lambda \approx w\frac{d}{L}$

A minor rearrangement provides the rule supplied in the instructions: $w \approx \frac{\lambda L}{d}$.

The three approximations made during the process of deriving the rule tell us that this rule cannot give exact results. However, for values of *w* in the micrometre range, the approximation is quite reasonable.